

Part 1: Basic Set Theory



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Basic Set Theory: Pillar of Semantic Modeling

- Naively, sets are just well-determined collections¹
 - Examples
 - Presidents of the United States
 - Letters of the alphabet
 - Failure modes of gas turbines
 - We will refer to such a collection as a “**class**”.
- The basic set relation is **membership**.
 - Symbol: \in
 - AKA *type*, “**is a**”
- Create a model containing the following declarations
 - PresidentOfUSA is a class.
 - GeorgeWashington is a member of this class.

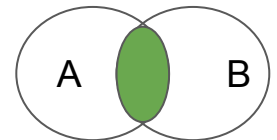
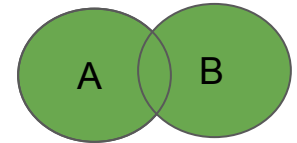
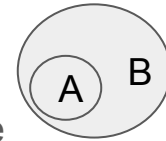


Solution

```
uri "http://sadl.org/BasicSetTheory1.sadl" alias bst1.  
PresidentOfUSA is a class .  
GeorgeWashington is a PresidentOfUSA.
```

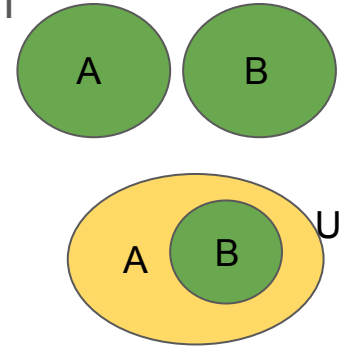
Set Operations

- Set $A \equiv$ (“***is the same as***”) set B if they have exactly the same members
- Subset: A is a subset of B if every member of A is also a member of B
 - Symbol: \subseteq
 - Also referred to as sub-class or “***is a type of***”
 - Proper subset: A is a proper subset of B if it is a subset but is not the same
- Union: the union of sets A and B is the set containing all members of A and all members of B
 - Symbol: \cup
 - Union is referred to by “***or***”, meaning “ $x \in (A \cup B)$ if $x \in A$ or $x \in B$ ”
- Intersection: the intersection of sets A and B is the set containing the elements in both A and B
 - Symbol: \cap
 - Intersection is referred to by “***and***”, meaning “ $x \in (A \cap B)$ if $x \in A$ and $x \in B$ ”



More about Sets

- Disjoint Sets: sets which cannot have members in common
 - “*A and B are disjoint.*”
 - “*{A, B} are disjoint.*”
- Compliment: within the universe U, A is everything not in B
 - “*{A, B} are types of U.*”
 - “*A is the same as not B.*”
- Extend/modify the previous model to contain the following:
 - MilitaryCommander is a class.
 - GeorgeWashington belongs to the intersection of PresidentOfUSA and MilitaryCommander.
 - BillClinton and HarryTruman belong to the PresidentOfUSA class.
 - Musician is the same as the union of the classes Singer and Instrumentalist.
 - WindInstrumentalist is a subclass of Instrumentalist.
 - Singer and WindInstrumentalist are disjoint.
 - Within the universe of Food, Vegetable is the complement of Meat.



Solution

```
uri "http://sadl.org/BasicSetTheory2.sadl" alias bst2.
```

```
PresidentOfUSA is a class .  
MilitaryCommander is a class.
```

```
GeorgeWashington is a {PresidentOfUSA and MilitaryCommander}.  
{BillClinton, HarryTruman } are instances of PresidentOfUSA.
```

```
Singer is a class.  
Instrumentalist is a class.  
Musician is the same as {Singer or Instrumentalist}.
```

```
WindInstrumentalist is a type of Instrumentalist.  
Singer and WindInstrumentalist are disjoint.
```

```
Food is a class.  
{Meat, Vegetable} are types of Food.  
Vegetable is the same as not Meat.
```

Additional Information

- Sets Can Be Partially Ordered into Hierarchies
 - Dog is a subset of Mammal
 - Mammal is a subset of Animal
 - Animal is a subset of LivingThing
 - LivingThing is a subset of PhysicalThing
- Two Sets of Interest
 - The set of all things (owl:Thing): every set is a subset of this set
 - The empty set, which has no elements: is a subset of every set
- Upper-level Ontologies
 - Lots of choices, see https://en.wikipedia.org/wiki/Upper_ontology.