Part 2: Logic

Andrew W. Crapo
Natural Semantics, LLC

Logic: the Second Pillar of Semantic Modeling

- Propositional Logic
    - A ***proposition*** is a statement which can be either *true* or *false*
        i. e.g., “it is raining (here, now)”
    - A proposition is often represented by a letter, often ‘*p*’ or ‘*q*’
    - There are 5 logical operators that can generate additional propositions
        i. Conjunction: ‘^’, ‘*&*’, ***‘and’***
            - Given *p* is *true* and *q* is *false*, what is ‘*p and q*’? (*false*)
            - Given *p* is *true* and *q* is *true*, what is ‘*p and q*’? (*true*)
        ii. Disjunction: ‘*v*’, ‘|’, ***‘or’***
            - Given *p* is *true* and *q* is *false*, what is ‘*p or q*’? (*true*)
            - Given *p* is *true* and *q* is *true*, what is ‘*p or q*’? (*true*)
        iii. Implication: ‘⟶’, ‘implies’, ‘***if … then...***’
            - Given *p* is *true* and *if p then q*, what can you say about *q*? (*true*)
            - Given *q* is *true* and *if p then q*, what can you say about *p*? (*nothing*)

Logic (cont.)

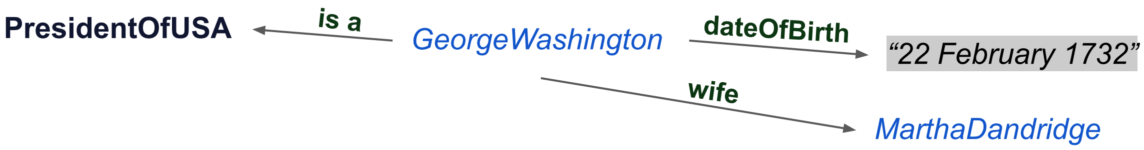
- Propositional Logic (cont.)
  - There are 5 logical operator (cont.)
    - iv. Necessary and sufficient conditions: ‘↔’, ‘***if and only if***’
      - Given *p* is *true* and *p if and only if q*, what can you say about *q*? (*true*)
      - Given *q* is *true* and *p if and only if q*, what can you say about *p*? (*true*)
    - v. Negation: ‘¬’, **‘*not*’**
      - Given *p* is *true*, what is *not p*? (false)
      - Given *p* is *true*, what is *not (not p)*? (true)

Logic (cont.)

- Predicate Logic, AKA First Order Logic
    - Predicate logic allows representation of the contents of a proposition
    - A ***predicate*** is a symbol which takes argument(s) and returns *true* or *false*
        - The number of arguments is called ***arity***
    - Predicates with arity 1 can represent class membership
        - *PresidentOfUSA(GeorgeWashington)* means *GeorgeWashington* ∊ *PresidentOfUSA*
    - Predicates with arity 2 can represent binary relationships between things
        - *wife(GeorgeWashington, MarthaDandridge)*
    - Predicates with arity 2 can also represent characteristics--data about something
        - *dateOfBirth(GeorgeWashington, "22 February 1732")*
    - Predicates with arity 1 can be transformed to arity 2
        - *PresidentOfUSA(GeorgeWashington) => is-a(GeorgeWashington, PresidentOfUSA)*

Logic (cont.)

- Predicate Logic, AKA First Order Logic (cont.)
  - Models with only arity-2 predicates are directed mathematical graphs and can be so visualized



PresidentOfUSA

is a

GeorgeWashington

dateOfBirth

“22 February 1732”

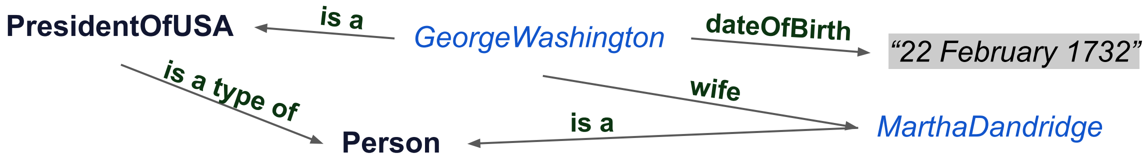
wife

MarthaDandridge

  - Each arity-2 predicate forms a ***triple*** with a ***subject***, ***predicate***, and ***object***
    - *predicate(subject, object) ≡ <subject> <predicate> <object>*
    - *wife(GeorgeWashington, MarthaDandridge) ≡ <GeorgeWashington> <wife> <MarthaDandridge>*
  - We will often refer to a predicate as a ***property***
  - A ***function*** is a symbol that takes 0 or more arguments and returns another non-logical symbol: not *true*, not *false*, not one of the 5 logical operators. More about functions later.

Logic Exercise

- Create a model equivalent to this visual graph



PresidentOfUSA

is a

GeorgeWashington

dateOfBirth

"22 February 1732"

is a type of

wife

Person

is a

MarthaDandridge

Solution

```
uri "http://sadl.org/Logic1.sadl" alias Logic1.

Person is a class.
PresidentOfUSA is a type of Person.

dateOfBirth is a property .
wife is a property.

GeorgeWashington is a PresidentOfUSA.
MarthaDandridge is a Person.

GeorgeWashington has dateOfBirth "22 February 1732", has wife MarthaDandridge.
```

Alternate Solution

```
uri "http://sadl.org/Logic1bb.sadl"alias logic1bb.

Person is a class.
PresidentOfUSA is a type of Person.

dateOfBirth is a property .
wife is a property.

GeorgeWashington is a PresidentOfUSA,
    with dateOfBirth "22 February 1732",
    with wife MarthaDandridge.

MarthaDandridge is a Person.
```

Things to Note

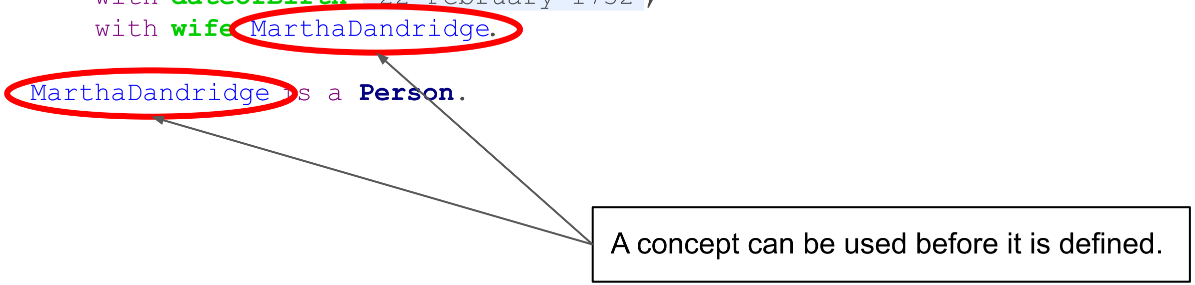
```
uri "http://sadl.org/Logic1bb.sadl"alias logic1bb.

Person is a class.
PresidentOfUSA is a type of Person.

dateOfBirth is a property .
wife is a property.

GeorgeWashington is a PresidentOfUSA,
    with dateOfBirth "22 February 1732",
    with wife MarthaDandridge.

MarthaDandridge is a Person.
```



A concept can be used before it is defined.

Quantification

- Quantifiers bind variables to non-variable symbols
    - Quantifiers allow us to make statements about multiple things declaratively
    - There are two quantifiers
- Existential quantifier
    - Symbol: $\exists$, read as “there exists”
    - Example: $\exists x: \mathrm{Person}(x) \wedge \mathrm{loves}(x, \mathrm{Logic})$
        - “There exists x such that x is a Person and x loves Logic” (“Someone loves logic”)
- Universal Quantifier
    - Symbol: $\forall$, read “for all”
    - Example: $\forall x: \mathrm{RichPerson}(x) \rightarrow \mathrm{loves}(x, \mathrm{Money})$
        - “For all x, if x is a RichPerson then x loves Money” (“All rich people love money”)
- Example with multiple quantifiers
    - $\exists x: (\mathrm{Man}(x) \wedge \forall y: (\mathrm{Man}(y) \rightarrow (\mathrm{shaves}(x,y) \leftrightarrow \neg \mathrm{shaves}(y,y))))$
        - “There is a man who shaves all and only men who do not shave themselves” (the barber paradox -- why is it a paradox?)[2]

Additional Information

- For a layman’s overview of logic see https://en.wikipedia.org/wiki/Logic.